Visualization of Symmetry in Graphs

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Abstract

Information can be modeled as graphs. *Graph Drawing* aims to construct geometric representations of graphs in two and three dimensions. Symmetry is one of the most important measures for good drawing. Further, symmetry is closely related to human understanding. We describe some methods for visualizing symmetries in graphs.

1 Introduction

Humans have a mechanism which automatically detects symmetry of objects. Thus, visualization of symmetry of objects enables human to recognize and understand the structure of the object easily.

Information can be modeled as graphs. *Graph Drawing* aims to construct geometric representations of graphs in two and three dimensions. Good drawings increase readability and understanding of the information modeled by the graph. Symmetry is one of the most important measures for good drawing.

However, visualization of symmetry of graphs is difficult. We need to solve three problems. The first step is to define a good model for symmetric drawing. The second step is to design algorithms to find symmetry in graphs. The last step is to display the symmetry, that is to construct a symmetric drawing of a graph.

The problem of drawing graphs symmetrically is NP-complete in general. However, for restricted classes of graphs, we are able to solve this problem in linear time.

We describe the method for visualizing symmetries in trees and series parallel digraphs.

2 Types of Symmetry

There are various types of symmetries in two and three dimensions. In two dimensions, there are two kinds of symmetry: a *rotation* by a point and a *reflection* by an axis.

However, symmetry in three dimensions is much richer than symmetry in two dimensions. The kinds are *rotation*, *reflection*, *inversion* and *rotary reflection*.

The difference from two dimensions is that a rotational symmetry in three dimensions is a rotation about an *axis*, and a reflectional symmetry in three dimensions is a reflection in a *plane*. Inversion is a reflection in a *point*. Rotary reflection is a composition of a rotation and a reflection.

3 Symmetric Drawing of Series Parallel Digraphs in Two Dimensions

Series parallel digraphs are one of the most common types of graphs: they appear in flow diagrams, dependency charts, and in PERT networks.

We can construct symmetric drawings of series parallel digraphs in two dimensions in linear time.

Series parallel digraphs have at most four types of symmetries in two dimensions: a horizontal reflection, a vertical reflection, a rotation by 180 degrees and a rotation by 360 degrees. First, we construct a canonical decomposition tree of a series parallel digraph and then label the tree. Then we find all possible symmetries using the labels. Finally we construct symmetric drawings of series parallel digraphs in two dimensions. Figure 1 shows an example.

4 Symmetric Drawing of Trees in Three Dimensions

Further, we extend symmetric drawings of graphs from two dimensions into three dimensions.

Trees have many applications. We construct symmetric drawings of trees in three dimensions in linear time.

First we define three dimensional symmetry model for trees: pyramid, prism and the Platonic solids. Then we construct the isomorphism class tree for efficient computation of symmetries in trees. Finally we find three dimensional symmetries in trees and then construct symmetric drawings of trees in three dimensions.

Figure 2 shows an example of the pyramid model and Figure 3 shows an example of the prism model. Figure 4 shows an example of the Platonic solid model.

5 Symmetric Drawing of Series Parallel Digraphs in Three Dimensions

Also, we can construct symmetric drawings of series parallel digraphs in three dimensions in linear time.

By using the canonical decomposition tree and the isomorphism class tree, we can construct symmetric drawings of series parallel digraphs in three dimensions.

Figure 5 shows rotational symmetry and Figure 6 shows reflectional symmetry. Figure 7 shows an inversion and Figure 8 shows all types of symmetry.

6 Conclusion

We describe the method for visualizing symmetries in trees and series parallel digraphs. Further, we would like to draw general graphs symmetrically.

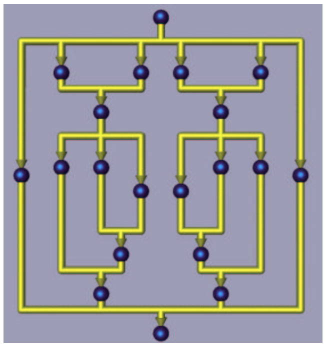

Figure 1: *Symmetric drawing of a series parallel digraph in two dimensions.*

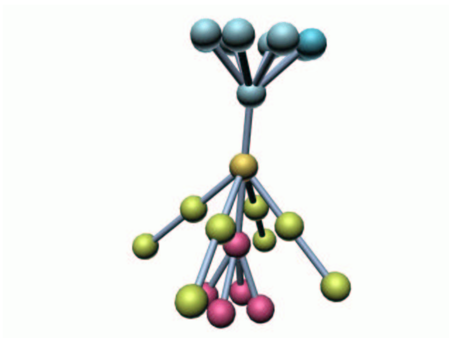

Figure 2: *The pyramid model.*

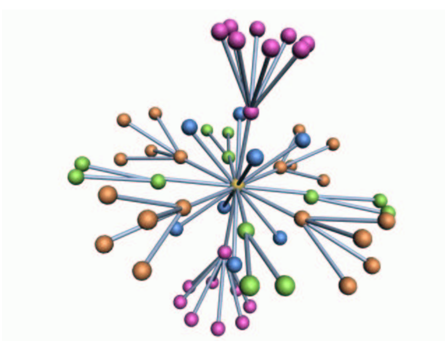

Figure 3: *The prism model.*

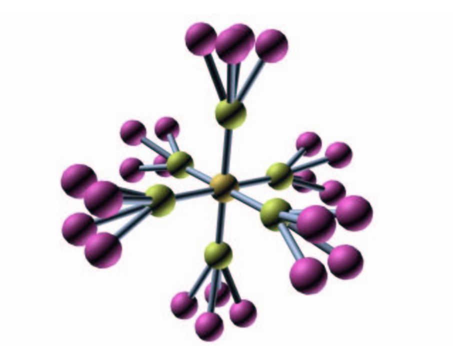

Figure 4: *The Platonic solid model.*

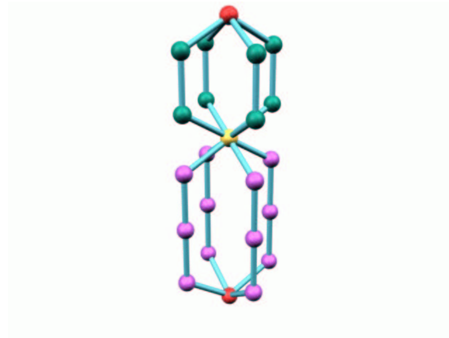

Figure 5: *Rotational symmetry of a series parallel digraph in three dimensions.*

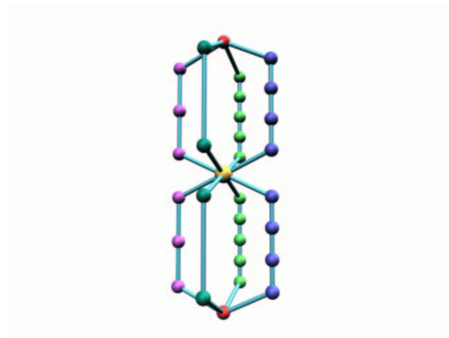

Figure 6: *Reflectional symmetry of a series parallel digraph in three dimensions.*

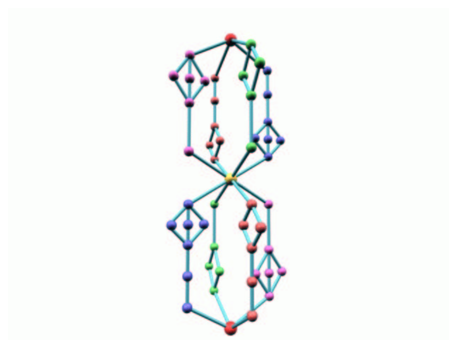

Figure 7: *Inversion of a series parallel digraph in three dimensions.*

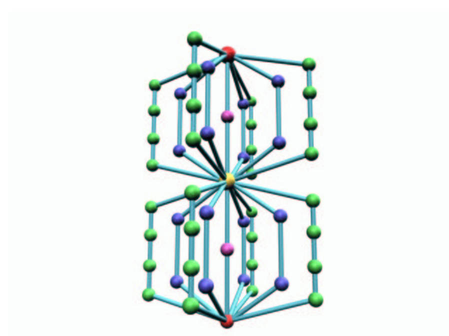

Figure 8: *Symmetric drawing of a series parallel digraph in three dimensions.*