An Effective Multilevel Thresholding Approach Using Conditional Probability Entropy and Genetic Algorithm

Yan Chang and Hong Yan

School of Electrical and Information Engineering
The University of Sydney, NSW 2006, Australia
Phone +61-2-93516659, Fax: +61-2-9351-3847

ychang@ee.usyd.edu.au

Abstract

Entropy-based image thresholding are used widely in image processing. Conventional methods are efficient in the case of bi-level thresholding. But they are very computationally time consuming when extended to multilevel thresholding since they exhaustively search the optimal thresholds to optimize the objective functions. In this paper, we propose a conditional probability entropy (CPE) based on Bayesian theory and employ Genetic Algorithm (GA) to maximize the CPE for the multi-thresholds. The experimental results show that CPE is a good criterion of image thresholding and GA is a applicable fast algorithm for multi-level thresholding compared to the exhaustive searching method.

1 Introduction

Image thresholding is an important first stage for many image processing applications. The entropy-based global thresholding approach has been concerning in recent two decades owing to its simple and straightforward property. In this approach, an image posteriori entropy first be defined in terms of its gray levels. For instance, Kapur's (1985) and Pun's (1980 and 1981) entropies based on the image histogram and Zheng's (1998), Zhao's (2001) and Fleury's (1996) entropies based on fuzzy partition. Secondly, a searching procedure is carried out to maximize the entropy in partition domain to determine the thresholded classes and optimal thresholds. For the bilevel thresholding , the search procedure is easy to carry out quickly. However, as pointed out in (Cheng 1999 and Fleury 1996), the computational complexity increases exponentially as the number of thresholds increases. For k classes, the searching times are up to $\frac{(L+k)!}{L!k!}$, where L denotes the number of gray levels. So a fast algorithm to find a maximum entropy in order to determine the optimal thresholds is necessary. Genetic Algorithms (GAs) have been found to have many advantages over traditional searching techniques (Yin 1999 and Goldberg 1989. One is that GA-based method is a global searching one and would not be trapped into local optimal solutions. Another important advantage is that the GA-based method can be faster by parallel implementation. In this paper, we propose a conditional probability entropy (CPE) based on Bayesian theory and employ Genetic Algorithm (GA) to maximize the CPE for the thresholds. CPE consider the fact that the pixels with the same gray level in an image may belong to different classes with different probabilities. An optimal classification for these pixels is to classify them to the class with higher probability. The chromosome structure in GA is designed based on the conditional probability function employed with two parameters.

2 Conditional Probability Entropy

Let D denote the two-dimensional intensity domain of an image I, and $G=\{0,1,\ldots L-1\}$ denote the L intensity values. Thus, an image I can be considered as a mapping from the two-dimensional domain D to the one-dimensional domain G.

Let $\mathrm{D}=\{D_k^* \mid k=1,2,\ldots,K\}$ denote K classes. The purpose of multi-level thresholding of an image is to classify its L classes in G into K classes in D. Due to the fact that the boundaries between the classes D_i^* and D_j^* ($i,j=1,2,\ldots K$ and $i \neq j$) are not well defined, some of the pixels with the same intensity value (i.e. corresponding to the same $g \in G$) may be classified into different classes. Therefore, it is assumed that for each $g \in G$, D_g is composed of K classes D_{kg} $(k=1,2,\ldots,K)$, which satisfy $D_g=\bigcup_{k=1}^{K} D_{kg}, D_{kg} \subset D_k^*$ and $D_{kg} \cap D_{jg}=\phi$ as $k \neq j$.

Let P_k^* denote the probability of the event D_k^*, i.e., $p_k^*=P(D_k^*)$. Based on Bayesian complete probability formula, we therefore have

$$p_k^*=\sum_{g=0}^{L-1} p_g \cdot p_{k|g}, \qquad k=1,2,\ldots K. \tag{1}$$

where

$$p_{k|g}=\frac{P(D_{kg})}{P(D_g)} \tag{2}$$

and

$$\sum_{k=1}^{K} p_{k|g}=1 \tag{3}$$

In above three equations, $p_{k|g}$ denotes the conditional probability of the event that a pixel is classified into the class D_k^* under the condition that the pixel has an intensity value g.

The information theoretic entropy measures the mean value of the uncertainty. In class space, entropy is the sum of the entropies in the K classes of D:

$$\begin{aligned} E(p_1^*, p_2^*, \ldots, p_K^*) &= -\frac{1}{\lg K}\sum_{k=1}^{K} p_k^* \lg p_k^* \\ &= -p_1^* \lg p_1^* - p_2^* \lg p_2^* - \ldots - p_K^* \lg p_K^* \end{aligned} \tag{4}$$

Thus, the entropy function E is a functional of p_k^*, ($k=1,2,\ldots K$). Since p_k^* is defined by the conditional probability functions $p_{k|g}$, $g=0,1,\ldots,L-1$, shown in Equation (1), entropy E given by Equation (6) is actually a functional of $p_{k|g}$.

The larger the value of $E(p_{1|g}, p_{2|g}, \ldots, p_{k|g})$, the more compatibility there is between h_g and $p_{k|g}$. Therefore, we assume that $p_{k|g}$ has the following forms:

for $0 < k \le (K-1)/2$

$$p_{k|g}(a_k, c_k) = \begin{cases} 1 & 0 \le g < a_k \\ f_k(g, a_k, c_k) & a_k \le g < c_k \\ 0 & c_k \le g < L \end{cases}, \qquad (5)$$

for $(K-1)/2 < k \le K-1$

$$p_{k|g}(a_k, c_k) = \begin{cases} 0 & 0 \le g < a_k \\ f_k'(g, a_k, c_k) & a_k \le g < c_k \\ 1 & c_k \le g < L \end{cases}, \qquad (6)$$

where $f_k(g, a_k, c_k)$ and $f_k'(g, a_k, c_k)$ is a monotonous decrease and increase continuous functions respectively.

Considering Equations (1), (5) and (6), the entropy function defined in Equation (4) is the function of $2K$ parameters $a_k, c_k, k = 1,2,\ldots,K$:

$$E(a_1, c_1, a_2, c_2, \ldots, a_k, c_k) =$$

$$\sum_{k=1}^{K} \sum_{g=0}^{L-1} h_g p_{k|g}(a_k, c_k) \lg\left(\sum_{g=0}^{L-1} h_g p_{k|g}(a_k, c_k) \right). \qquad (7)$$

A set of optimal parameters $(\tilde{a}_k, \tilde{c}_k \mid k = 1,2,\ldots,K)$ should be the one where Equation (7) has a maximum. The optimal thresholds can be obtained by $\tilde{t}_k = \dfrac{\tilde{a}_k + \tilde{c}_k}{2}$.

3 Optimal Thresholds' Search Using Genetic Algorithm

Genetic Algorithms (GAs) are known to be robust (Goldberg 1989) and have enjoyed increasing popularity in the field of numerical optimization in recent years. GAs are search algorithms based on the mechanics of natural selection and natural genetics.

The following four parts are generally involved in a GA searching for any problems:

1. An effective chromosome encoding method
2. A fitness function to be maximized or minimized
3. A selection procedure to select the pairs with better genes for producing offspring
4. A mating process to produce offspring from their parents

Five parameters are considered in proposed GA: (1) population size S_{pop}; (2) crossover probability Pc; (3) mutation probability Pm; (4) the total number of bits in a generation Nb; (5) maximum number of generations Ng.

To map our search problem to GA, the following four aspects corresponding to the above four parts are determined.

3.1 Chromosome Encoding

The $2K$ parameters consist of a chromosome v that has the order of $(a_1, c_1, a_2, c_2, \ldots, a_K, c_K)$. The population with S_{pop} chromosomes can be described as

$$v_j = (a_1^j, c_1^j, a_2^j, c_2^j, \ldots, a_K^j, c_K^j) \quad j = 1,2,\ldots,S_{pop}$$

where
$$\begin{cases} a_i^k \le c_i^k \\ a_i^k < a_j^k, \ \forall i < j, \quad k = 1,2,\ldots,K \end{cases} \qquad (8)$$

Each parameter has ($\log_2 L$) bits. So a chromosome has ($2K \log_2 L$) bits. The construct of a chromosome is shown in Figure 1.

a_1	c_1	a_2	c_2	…	a_i	c_i	…	a_K	c_K

Figure1. One chromosome structure

For randomly generating v_j, it is generally possible that Equation (8) is not sufficient. Here we transfer the insufficient parameters from the range $[0,L-1]$ to the following ranges in order to meet the condition of Equation (8)

(a) if $a_i \ge a_j$, transfer a_i to $[0, a_j]$ by calculating $a_i'^k = \dfrac{a_i^k a_j^k}{L-1}$.

(b) if $a_i \ge c_i$, transfer c_i to $[a_i, L-1]$ by calculating $c_i'^k = c_i^k \left(1 - \dfrac{a_i^k}{L-1} \right)$.

3.2 Fitness Function

For the problem of our image thresholding method, the fitness function $E(v)$ is the entropy given by the Equation (7).

3.3 Selection Procedure

This procedure is to select the pairs for producing their offspring with respect to the probability distribution P_s^j, $j=1,2,\ldots,S_{pop}$, based on the fitness value. We employ the selection procedure based on spinning a roulette wheel S_{pop} times (Goldberg 1989). A Cumulative probability for each chromosome v_j is

$$P_s^j = \sum_{i=1}^{j} E(v_i)/F \quad , j=1,2,\ldots,S_{pop}.$$

where $F = \sum_{i=1}^{Spop} E(v_i)$ is the total fitness of the population.

Generating a random number r from the range [0,1], if $r < P_s^1$, then select the first chromosome v_1; otherwise select the j^{th} chromosome v_j $(2 \le j \le S_{pop})$ such that $P_s^{j-1} < r \le P_s^j$.

3.4 Mating Process

After several pairs are selected as parents randomly with probability P_s, the mating process is carried out. In this process, it is hoped that the selected parents combine their good characteristics to produce a better offspring. It is accomplished by GAs through crossover and mutation operators.

- *Crossover operator*: first generate a random number in [*1, 16K*] as the position of the crossing point of the parents' chromosomes. Next, swap the parents' genes after the crossing point to produce their offspring.
- *Mutation operator*: this is performed on a bit-by-bit basis. There are $N_b = 2KSpop \log_2 L$ bits in a population, so generate N_b random numbers from the range [0,1]; if the random number is less than the predefined mutation probability P_m, mutate the bit.

Till now, we have obtained a new population (generation). This procedure goes on till a preset generation number *Ng* is reached at.

In conclusion, the procedure of the proposed GA search is as follows:

Step1: Initiation: random generate an initial population V_1 which consists of S_{pop} chromosomes.

Step2: Evaluation: evaluate fitness function for all chromosomes by Equation (7)

Step3: Selection: use roulette wheel to select new population V_k with respect to the probability distribution based on fitness value.

Step4: Crossover:

- select chromosome pairs as parents for crossover with respect to the probability P_c
- determine the cross point using a random integer in $[1, 2(K-1)\log_2 L]$
- mate parents with each other to generate their two children
- replace parents by their children for a new population

Step5: Mutation:

- generate $16KS_{pop}$ random number in $[0,1]$
- if $r<P_m$, mutate the corresponding bit, where P_m is the mutation probability .

Step6: Termination: if generation iterations equal *Ng* , stop the GA.

4 Experimental Results

We applied the proposed method to many kinds of images. The experiment results for four of the images,

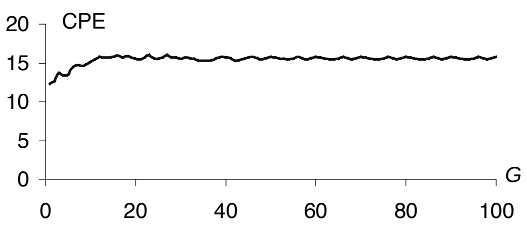
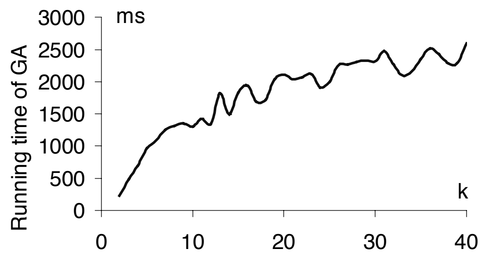


Figure 2. CPE Vs generations in GA on average

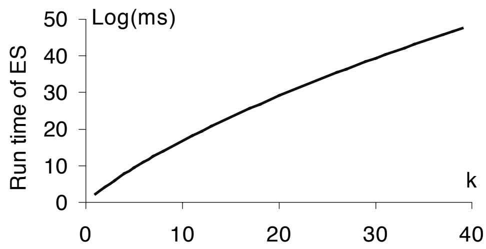
Family, Computer, Model and Ball, with 256 gray levels are presented in this section. Figure 2a and 3a show two original images: *Family* and *Model*. Since GAs are stochastic, we run each GA ten times and evaluate their average results. In our experiment, the population size S_{pop} is 30 and the probabilities of a cross-over P_c and a mutation P_m are set to 0.6 and 0.1 respectively.

Figure 2 is the maximum CPE graph in generations on average of four images. It is observed that the GA converges very fast in 10-20 generations. Thus, the number of generations is selected to 20 in this paper.

Figure 3b-3d and 4b-4d present the thresholding results for the two images with three-, five- and eight-level respectively. For *Family* image, the optimal thresholds obtained by CPE and GA searching method are (32,86), (18,40,75,128) and (13,23,39,60,83,122,169) with three-, five- and eight-level respectively. While for *Model* image, the optimal thresholds are (132,230), (78,146,195,240) and (45,98,140,198,220,232,239) with three-, five- and eight-level respectively.



(a)



(b)

Figure 5 Searching times with different *K*

For demonstrating the efficient of proposed approach, the running times of multilevel thresholding using GA and exhausted searching (ES) method (Pun 1980 and Cheng 1998) are compared in Figure 5. It is proved that the running time of the exhausted search (ES) (figure 5a) increase exponentially as the number of thresholds increases like mentioned in Section 1. The running time, however, increase rough linearly using GA search (Figure 5b) with class number increases.

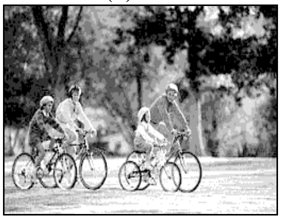
5 Conclusion

In this paper, it has been developed a thresholding algorithm based on conditional probability entropy

(a)

(b)

(c)



(d)

Figure 3. *Family* (a) original image (b-d) thresholed images with 3-,5- and 8-level respectively

(CPE). For obtaining optimal thresholds, a GA is

D. E. designed and applied to speed up the optimal searching procedure. Experiments shows that CPE with GA search are effective and much faster than CPE with conventional exhausted searching (ES) method.

6 References

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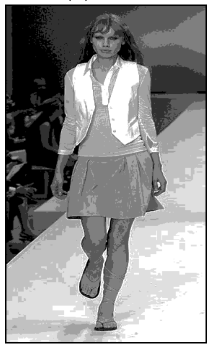
(a)

(b)

(c)



(d)

Figure 4. *Model* (a) original image (b-d) thresholed images with 3-,5- and 8-level respectively

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