Evolutionary Design of Optical Waveguide with Multiple Objectives

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Abstract

In this paper, we investigate a real-world problem, constructing optical waveguide structures using evolutionary search strategies. Optical waveguide is the most basic component in optical communication and integrated optical circuits. The structure of a waveguide is of great importance as it would significantly impact on the quality of light transmission. The aim of this paper is to find a set of potential structures which satisfy multiple waveguide design objectives including minimum group velocity dispersion and minimum propagation loss. Therefore, evolutionary algorithms which are populate-based search techniques are more suitable for this type of tasks. As a part of this investigation, a GP-based parametric optimization methodology called Parameter Mapping Approach (PMA) is introduced. This method together with traditional GA have been adapted into this study. The experiment results demonstrate both PMA and GA can produce multiple waveguide structures that meet the design criteria. Furthermore these evolved structures have very low dispersion and loss compared to those reported in the current literature.

Keywords: Optical Waveguide, Genetic Programming, Genetic Algorithms, Parameter Optimization, Structural Optimization

1 Introduction

Optical signal transmission is a foundation of our modern communication networks. Comparing with traditional electronic signal transmission, optical methods have more advantages such as more energy efficient less interference and higher capacity in carrying information. In these optical transmission networks the user data are aggregated and converted into optical signal to be transmitted in optical fibers. At the central office or switches, the optical signal is converted back to electrical domain so that the data can be regenerated, buffered or switched. The data stream are then converted back to optical signal for next transmission. Although optical fiber can transmit data at very high rate, processing data in the current approach in the electrical domain is the main bottle-neck of the current optical transmission systems. As the demand for bit-rate continues to growth, it is desirable to process the optical signal directly in the optical domain. All optical signal processing is possible by ultilising the nonlinear properties of the optical transmission medium such as optical fibers or optical waveguides. Optical waveguides are the preferred medium for all optical signal processing since they allow for compact devices and the possibility of integrating many functions into a single device to create an integrated photonic chip. By using waveguide meterials with strong optical nonlinearity such as Chalcolgnise, many optical signal processing functions can be realized on a short optical waveguide (Eggleton et al. 2011). In order to achieve highly efficient optical signal processing functions in a waveguide, it is important to minimize the propagation loss and the dispersion of the waveguide mode. By optimizing the waveguide structure, it is possible to achieve both low loss and low dispersion waveguides.

Structural optimization is an important area itself. In many circumstances, the complexity of the problem is high because there are a large number of factors to be optimized such as shape, quality and dimension. Moreover often there are many constrains to be taken into consideration, such as weight, size, cost and so on. This type of optimization tasks can be divided into two categories as suggested by (Rasheed 1998), pure structural optimization and parametric optimization. The former one involves making high level decision about geometric properties of the structure while the latter mainly focuses on the numeric aspects of structures, that is finding more suitable combinations of parameters for a given shape. Waveguide structure design can be addressed at both levels. The study presented here only focuses on the latter, finding better parameters for single-ridge waveguides.

One prominent approach in optimization is evolutionary search methods which are inspired by Darwin's natural selection principle. Among a population of potential solutions, the better ones are selected to create a new generation of solutions which presumably will be better than the previous generation. This iterative process usually stops at a point that a good solution is found or no improvement can be achieved. The main methods under this category include Genetic Algorithm (GA) (Holland 1975), Genetic Programming (GP) (Koza 1992), Differential Evolution (DE) and Particle Swarm Optimization (PSO). Because evolutionary methods are population based, they are capable of provide not only one solution but a set of solutions. Finding multiple solutions is one the aims of our waveguide design task.

GA has been successfully used in a wide range of parametric optimization problems (Pujol & Poli 2004*a*) and remains as a main choice for this kind of tasks. Therefore GA is selected for this study. In addition GP is also introduced here although it is not strong in parameter optimzation but strutrual optimization such as designing circuits and satellite antenna (Lohn et al. 2004). Becuase our long term goal is to include strucutal optimization for waveguide desgin as well. The conventional symmetric single ridge waveguide is not necessarily the best structure. Ideally one method, presumably a GP-based technique, can optimize both the structure as well as its parameters. Therefore GP is used and compared with GA. As proposed and studied by Pujol & Poli (2008), GP could be adapted into parametric optimization and even outperform others in some cases when applying to some benchmark functions (Ingber & Rosen 1992).

Another aspect of this study is its multiple objectives. The quality of a waveguide structure is not measured by just one criterion. Multiple objectives such as minimum dispersion and minimum propagation loss are all highly desirable in waveguide applications. This is another reason of using evolutionary methods as they can be naturally adapted into multi-objective optimization (Deb et al. 2002, Zitzler et al. 2001).

The rest of this paper is organized as such: Section 2 explains the basics of waveguide structure and the meanings of dispersion and propagation loss which we mentioned above. This section also covers a brief introduction to GP/GA and the related work. Section 3 presents the methodology used in this study. Section 4 describes the experiments with the results. Sections 5 concludes this study and discusses our future investigations.

2 Background

The first two parts of this section introduces the basics of optical waveguide and GP, GA briefly. Additionally the related work for structural design, parametric optimization as well as waveguide structure optimization are reviewed.

2.1 Optical Waveguide

Optical waveguide is a medium to guide light wave propagation. In this paper a single-ridge waveguide structure is considered. Figure 1 shows a modeled structure of optical ridge waveguide, which is the basis of this study. The middle core layer is one kind of chalcogenide glasses As_2S_3, a highly nonlinear crystalline material. The properties of the waveguide mode are determined by the waveguide cross-section parameters , including the ridge width, the height of the core layer and the etch depth.

The measurement of waveguide structure is a key point in our experiment. In industry, the most accurate way is to produce a real waveguide and test it through some devices. However, this operation often time consuming and costly. The alternative is using simulators. The simulator in this study is from RMIT's Microplatform Research Group and has been used for a series of waveguide projects (Nguyen et al. 2009*a*,*b*).

There are a set of properties which are related to the quality of waveguides, such as single-mode or multi-mode, dispersion, nonlinearity, loss and etc. This study concerns two properties: the dispersion and the loss. The followings are their descriptions.

- *Dispersion:* Group velocity dispersion, or simply dispersion in this study, of a waveguide mode is a parameter that measures how the group velocity of the waveguide mode depends on wavelength of frequency. Group velocity dispersion is caused by material dispersion and waveguide dispersion. The material dispersion comes from material. Waveguide dispersion is caused by the geometric (structural) reasons. As indicated in prior literature, in some certain situations these two kinds of dispersion can compensate each other and result in a zero-dispersion waveguide (Lamont et al. 2007).

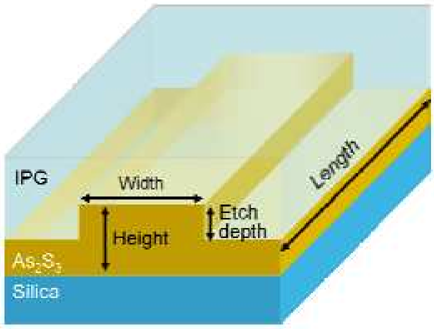

Figure 1: A modelled structure of As_2S_3 waveguide. (Courtesy to M. R. E. Lamont, etc.)

- *Propagation loss:* The propagation loss of a waveguide mode can be caused by material absorption, scattering loss and leakage loss. At telecommunication wavelength of 1.55 μm, the materials used in the considered Chalcognide waveguide have negligible absorption loss. The scattering loss is mainly determined by the fabrication process. Leakage loss is caused by the coupling between the guided mode and radiation modes of a waveguide. Leakage loss can be effectively reduced to zero by optimizing the waveguide structure (Nguyen et al. 2009*c*). In this study, material loss and scattering loss are ignored when consider waveguide propagation loss.

It should be noted that the loss is measured as dB/cm, decibel per centimeter of the waveguide. The aim is to find waveguide structures with both zero-dispersion and low loss. In fact absolute zero-dispersion is hardly achievable at telecommunication wavelength of 1.55 µm. Instead the following formula is used to define zero-dispersion.

$$Dispersion = |\frac{\partial^2\beta}{\partial\omega^2}| < 1.0 \times 10^{-26}(nm^2/m)$$

The partial second derivative part $|\frac{\partial^2\beta}{\partial\omega^2}|$ is the wave equation in which β is a function of ω, angular frequency. Note for legibility, zero dispersion can be also expressed as:

$$|\frac{\partial^2\beta}{\partial\omega^2} \times 10^{24}| < 0.01(nm^2/m).$$

Additionally, there is another criterion which is used to determine zero-dispersion: whether the waveguide is able to achieve absolute zero dispersion at around 1.55 µm wavelength. It is believed in such case we can shift the zero-dispersion wavelength to telecom wavelength (Lamont et al. 2007). In order to validate this criterion, we should plot all the dispersion value from 1.4 µm to 1.7 µm wavelength. Figure 2

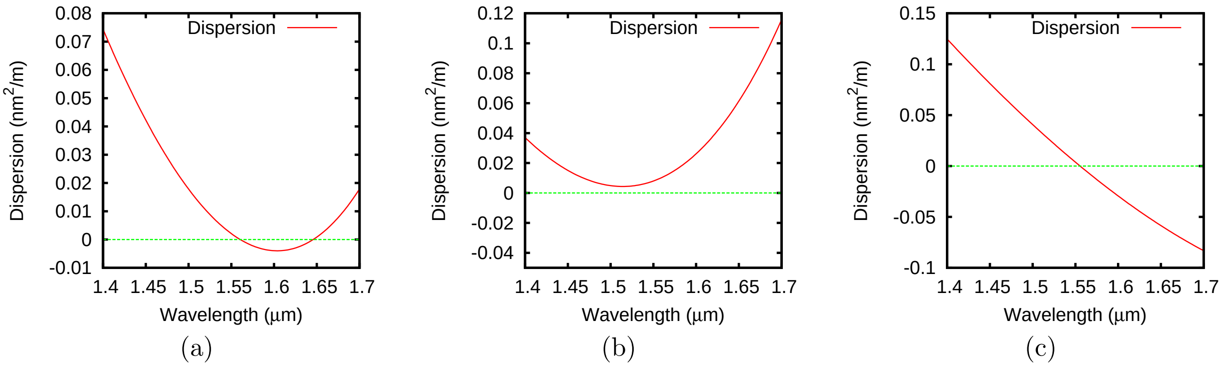

Figure 2: Three different kinds of distribution for dispersion property

shows three possible dispersion distributions. All of them meet the first requirement which can have dispersion value less than 0.01 nm^2/m. However, the case in Figure 2 (b) is not valid as it is not able to achieve absolute zero-dispersion at 1.55 µm wavelength.

In terms of propagation loss, recent research reported structures with loss down to $0.2dB/cm$ (Madden et al. 2007, Cardenas et al. 2009, Ruan et al. 2005) as that is considered good quality in the field. However we aim to further reduce the loss below $0.1dB/cm$. Propagation loss is calculated as following formula:

$$Loss = 8.868 \cdot \alpha \cdot \frac{2\pi}{\lambda \cdot 100} < 0.1(dB/cm)$$

The λ value in the formula is the wavelength of optical transmission. It is set as 1.55µm which is mentioned before, the commercial telecommunication wavelength. The α value is a parameter to indicate reduction in light density. It can be produced by waveguide simulator. Thus the aim of reducing loss below $0.1dB/cm$ is equivalent to bringing down the α value to a low level: $\alpha < 2.8 \times 10^{-7}$.

2.2 Genetic Algorithm and Genetic Programming

Genetic Algorithm (GA) and Genetic Programming (GP) are two typical members in the area of Evolutionary Computing. GP could be considered as a variation of GA as they do share large amount of similarities.

Both GA and GP randomly generate a population of solutions as the initial generation. These solutions, also called individuals, are then evaluated in terms of their capability in solving a particular problem. The better ones have higher probabilities of being selected for creating the new generation. Therefore the next generation is likely better than the previous one. Majority of individuals in the new generation are created by exchanging genetic materials among parents, the individuals selected from the previous generation. This process is known as crossover. Some individuals are created by randomly changing one selected parent. This process is named mutation. Some individuals are just straight copied from the previous generation. This is called elitism.

An evolutionary process will continue from generation to generation until a perfect solution is found or one of other criteria is met such as no improvement for a number of generations, or a maximum number of generations is reached. The driving force of this evolutionary process is the fitness measure which determines the survivability of individuals. The fitness tends to improve over the generations. In our case, the fitness is the quality of a waveguide structure, in terms of both zero dispersion and low propagation loss.

The main difference between GA and GP is how to represent an individual. In GA, an individual is often a fixed-length binary string (chromosome) which can naturally be used to express a list of numeric parameters. By crossing over between different individuals or mutating one individual, various combinations of parameters can be created. The better ones will survive and eventually emerge as the final solution. Due to this linear representation GA is suitable for parametric optimization of which the number of parameters are given.

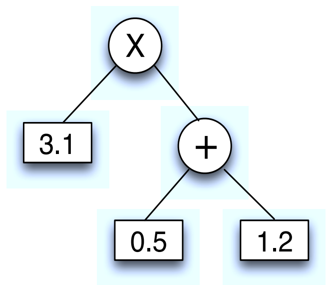

Figure 3: GP tree representation

In contrast an individual in GP is often represented as a tree as shown in Figure 3. The internal nodes on a tree are called functions which are often some kind of operators and the leaf nodes are called terminals which often are input values or parameters for the function nodes connected to them. The tree shape is usually very flexible as long as the number of levels on the tree is within a limit and the tree is syntactically sound, meaning it can be evaluated without any error. Crossover in GP is done by two parents swapping tree branches. Mutation in GP is randomly replacing one branch on an individual with an external branch. It can be seen that new individuals would be very different to their parents topologically. Due to this tree representation GP is suitable for exploring different structures. GP has shown its ability and effectiveness in designing structures and solving wide range of real-world problems(Poli et al. 2008).

2.3 Related Work

In the literature waveguide structures are optimized by domain experts from the area of photonics. Lamont et al. (Lamont et al. 2007) proposed a dispersion engineering method of As_2S_3 waveguide structure with dispersion $\approx 0.24nm^2/m$ and loss value $\approx 0.25dB/cm$ in 2007. Cardenas et al. demonstrated an optical waveguide structure with propagation loss of 0.3dB/cm at 1.55µm using Silicon as the medium material(Cardenas et al. 2009). Madden et al. discovered a structure with loss as low as 0.05dB/cm at telecom wavelength(Madden et al. 2007). All these structures were optimized manually with domain knowledge. Our aim is to generate a set of better waveguides without requiring domain experience and human intervention during the process.

Structural optimization is an important area in GA. (Rasheed 1998) attempted to optimize the aircraft structure using GA. In their work, the structural optimization is actually treated as optimizing several parameters. The task is to determine the dimensions of the aircraft, the length of wings and so on, while the basic shape of the aircraft does not change. (Chafekar et al. 2003) continued the previous work and improved its performance in addressing constrained multi-objective optimization problem. In the aircraft design, the structural optimization problem is converted into a parametric optimization problem. This strategy is applied in this study.

GP, as a strong problem solving method, has demonstrated its capability in designing structures topologically. NASA uses GP to design satellite antennas. Their investigation produced an antenna with higher ratio of signal gain to self weight. It is more effective than the designs from NASA engineers. Also, it was stated that the GP design schema significantly reduced the design life cycle (Lohn et al. 2004).

3 Methodology

The waveguide design problem is addressed by two evolutionary methods: using the relative new PMA technique and using the traditional GA. Firstly we explain the PMA methodology and then give a brief introduction of Non-dominated Sorting for multi-objective optimization.

3.1 GA and PMA methodology

For a single-ridge symmetric waveguide, there are three variables, the width, the height and the etch depth as illustrated in Figure 1. As the basic shape does not change, the task is actually a parametric optimization problem. The representation for GA is straightforward, three double numbers expressed as one binary string. The first part of the string corresponds to width, the middle part to the height and the last part for the etch depth.

For GP the representation is a little more involved as parametric optimization is not native for GP. In order to achieve this, Parameter Mapping Approach (PMA) is used (Pujol & Poli 2004*b*). The basic idea is not directly finding a good combination of parameters, but searching for GP individuals as a mapping function which can accept a set of raw inputs to produce another set of adaptive parameters. The optimal combination of parameters are searched indirectly.

Figure 4 shows the procedure to evaluate an individual in PMA. A mapping function receives a set of raw parameters as input and transforms them to three parameters which are interpreted as the width, the height and the etch depth of a waveguide. Note that, during the evolutionary process, all individuals share the same raw parameters. That is different to the original PMA proposed in (Pujol & Poli 2004*b*), in which the initial raw parameters are randomly generated. By introducing a constant initial parameter, these generated mapping functions could share some of the components or building blocks. Moreover the step of generating random numbers for each raw parameter can be removed.

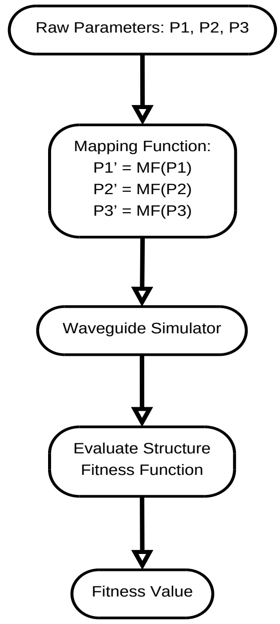

Figure 4: Fitness evaluation for a GP individual

The evaluation of a combination of parameters is done by a waveguide simulator developed by Microplatform Research Group in RMIT. It can simulate the transmission process in a given waveguide structure and therefore calculate its properties such as dispersion and propagation loss. These output from the simulator are used to assign fitness values of individuals either in GA or GP.

PMA uses 4-arity operators as the GP functions, because they can break the symmetry of the addition and multiplication arithmetic operators (Pujol & Poli 2004*b*), thus reducing the possibility of the so-called *permutation* or *competing convention* problem (Radcliffe 1991, Hancock 1992).

Table 1: GP Functions used in PMA

Plus	$x \times y + u \times v$
Minus	$x \times y - u \times v$
Multiply	$(x + y) \times (u + v)$
Divide	$PDV(x + y, u + v)$

The function set of GP is listed in Table 1: where $PDV(num, den)$ is the protected division, which returns num if the denominator den is zero. These four are the only operators used. The terminal set is simply random numbers and raw input. Tournament selection strategy is employed in this model. The GA and GP runtime parameters are described along with the experiments.

To evaluate the robustness of PMA, a serials of test functions were introduced. The first step of optimizing waveguide structures is using only one objective, dispersion which is the most important measure for waveguides. Both GA and GP were used for this single objective task. Followed the single objective experiments, both dispersion and loss were introduced as the objectives for GA and GP.

3.2 Non-dominated Sorting/NSGA-II

Non-dominated Sorting Genetic Algorithm (NSGA) was first proposed by (Deb et al. 2002). Since the time it was developed, this algorithm has been criticized due to its high computational complexity of nondominated sorting, lack of elitism. Further investigation on this approach leads to an improved version of NSGA, namely *Non-dominated Sorting Genetic Algorithm II.* Instead of only one optimal solution, the NSGA-II provides a set of optimal solutions. The multiple solutions are those none of which that can be considered to be better than any others with respect to all objectives. This set of optimal solutions is known as a *Pareto Optimal Set* or a *Pareto Frontier.*

For a multiple objective optimization problem, a feasible solution can be represented as a vector X:

$$X(Objective1, Objective2, ..., ObjectiveN)$$

This solution is considered to be non-dominated if and only if,

- For any other vector Y, each objective determined by vector X is better or at least equal to that one determined by vector Y.
- For any other vector Y, at least one of the objectives determined by vector X is strictly better than the corresponding objective determined by vector Y.

For a given number of solutions, there is only one vector that can satisfy both the above criteria. It cannot be improved without worsening at least another objective. The *Pareto Optimal Set* is composed of such kind of solutions.

In terms of the implementation of NSGA-II algorithm, the basic idea is to divide the population into a number of sub-populations referred as fronts which are ranked in terms of levels (Nguyen & Yousefi 2010). For each front, there is one non-dominated solution which satisfies the above two criteria. In this way, for the entire population, there is a set of non-dominated solutions derived from the individual frontier.

In the second generation starting from the initial population, these ranked points are then reproduced through genetic operators. Individual elements with a higher rank are more likely to be selected for reproduction. The solutions in the first level front are assigned the highest priority, and then are those in the second level and so forth. Eventually the Pareto frontier is formed as the rank can no longer be improved.

4 Experiments

Three groups of experiments mentioned above are presented in this section. The difficulty gradually increases.

4.1 Optimizing Test Formulae

The task there is to find the minimum of the following four formulae of which the number of parameters differs. The fitness measure is the lower output the better.

- $f_1(x, y) = x^2 + y^2$
- $f_2(x, y) = 100 \times (x^2 - y)^2 + (1 - x)^2$
- $f_3(x, y, z) = (x - 5)^2 + (y - 15)^2 + (z - 40)^2$
- $f_4(a, b, c, d, e) = (a+0.5)^2+(b-55)^2+(c-0.5)^2+(d - 15)^2 + (e - 99)^2$

Table 2: Test Formulae for PMA

Formula	No. of Parameters	Success Rate
$f_1(x, y)$	2	99/100
$f_2(x, y)$	2	99/100
$f_3(x, y, z)$	3	100/100
$f_4(a, b, c, d, e)$	5	97/100

The population size here was set to 200 and the total number of generations is 100. The GP runtime configuration was that 80% crossover rate, 10% mutation rate and 10% elitism.

Table 2 shows the results: the number of runs (out of 100 runs) can find a solution of which the output is lower than 0.001. Absolute zero was not required here as it was the case in the waveguide design. The success rate was very high in all the experiments. The modified PMA method is capable in parametric optimization on simple tasks.

4.2 Optimization with Single Objective: Dispersion

The goal of this set of experiments is to search for structures with zero-dispersion. There is only one objective. For each of the parameter, there is a reasonable range of values. Parameters outside of this range are not practical therefore should not be explored. The ranges are

$$waveguide\ width \in [0.1\text{µm}, 2.0\text{µm}] \quad (1)$$
$$waveguide\ height \in [0.1\text{µm}, 1.5\text{µm}] \quad (2)$$
$$etch\ depth \in [0.0\text{µm}, waveguide\ height] \quad (3)$$

As GA represents numerical values of these parameters directly, so the range can be easily imposed on these parameters in GA. However, the PMA methodology does not deal with values directly. Thus the value for an individual parameter can not be guaranteed to situate in that range setting. Therefore a parameter normalization procedure is introduced here. It can be expressed as:

$$Parameter = LOW + \frac{UP - LOW}{1 + |OUTPUT|}$$

where:
LOW is the lower limit of the parameter;
UP is the upper limit of the parameter;
OUTPUT is the output value from a GP tree.

Note that the third parameter etch depth is can not be larger than the second parameter waveguide height. To fulfil this constrain, a *swap* strategy is introduced which examines all individuals before fitness evaluation, both for GA and for PMA. If one individual violates that constrain, then its third parameter will be swapped with the second parameter. If its third parameter is beyond the up limit, 1.5µm, then it will be trimmed to 1.5.

In terms of the fitness measure for GA and PMA, they are the same. As there is only one objective, the fitness evaluation can be simply expressed as:

$$fitness = |Dispersion|$$

Table 3: Optimizing Dispersion in TM mode by PMA and GA

Solutions	Width(µm)	Height(µm)	Etch Depth(µm)	Dispersion (nm^2/m)	Approach
1	1.563	1.092	0.442	0	PMA
2	1.059	1.043	0.991	0	PMA
3	1.042	1.065	1.011	0	PMA
4	1.089	1.083	1.066	0	PMA
5	1.386	1.240	1.180	0	PMA
6	0.482	1.172	1.122	0	PMA
7	0.842	0.828	0.742	0	PMA
8	1.253	1.087	0.847	0	PMA
9	0.584	0.717	0.657	0	PMA
10	0.732	0.732	0.632	0	PMA
11	1.576	1.091	0.439	0	GA
12	1.670	1.422	0.699	0	GA
13	1.576	1.077	0.759	0	GA
14	1.338	1.177	0.873	0	GA
15	1.144	1.018	0.393	0	GA
16	1.803	1.132	1.032	0	GA
17	1.555	0.988	0.343	0	GA
18	1.576	1.091	0.439	0	GA
19	1.103	1.091	1.028	0	GA
20	0.937	1.091	0.747	0	GA

Table 4: Optimizing Dispersion in TE mode by PMA and GA

Solutions	Width(µm)	Height(µm)	Etch Depth(µm)	Dispersion (nm^2/m)	Approach
1	0.943	0.943	0.843	0	PMA
2	1.339	0.807	0.798	0	PMA
3	0.946	0.946	0.846	0	PMA
4	0.965	1.247	1.185	0	PMA
5	1.355	1.331	1.260	0	PMA
6	0.778	0.931	0.874	0	PMA
7	0.935	0.935	0.835	0	PMA
8	1.311	1.383	1.329	0	PMA
9	0.500	0.647	0.569	0	PMA
10	1.320	1.320	1.220	0	PMA
11	1.978	1.266	1.010	0	GA
12	1.250	1.341	1.224	0	GA
13	1.250	1.175	1.021	0	GA
14	1.250	1.233	1.224	0	GA
15	1.250	0.998	0.992	0	GA
16	0.620	1.335	1.224	0	GA
17	1.978	1.266	1.010	0	GA
18	1.250	1.175	1.021	0	GA
19	0.620	1.335	1.224	0	GA
20	1.250	1.175	1.021	0	GA

There is another issue related to the optimization process, that is the multi-modes waveguide dispersion measurement. As in this set of experiment, the *single mode* condition was not take into consideration, it is possible to work out waveguide structures with multiple modes. For each mode in waveguide, it has its corresponding dispersion and loss value. Regarding to this situation, average value is usually expected. However, this strategy is not satisfactory here, because in real-world optical applications, only the mode with the lowest dispersion value will be used. Thus, we selected the lowest dispersion value for the fitness measurement of multi-modes waveguide.

The run time configuration for GA and PMA were the same: 80% of crossover, 10% of mutation and 10% of elitism. The population size and maximum number of generations were also same, which were both 200. In PMA, the maximum depth of the GP tree is set to 5 to avoid solutions being too complex.

There are two main transverse modes need to be considered in waveguide design, TE (Transverse Electric) mode where there is no electric field along the propagation path, and TM (Transverse Magnetic) mode where there is no magnetic field along the propagation path. In this study we investigate both TM mode and TE mode. In optical applications, the waveguide will be applied either in TM environment or TE environment.The results for TM mode and TE mode are shown in Table 3 and Table 4 respectively.

The solutions listed in Tables 3 and 4 are the best individuals found by these evolutionary search processes. It can be seen that most of them are different. That means both GA and GP are capable of finding multiple solutions. This feature is important in practice as engineers would have choices in deciding which one to use under different circumstances, such as manufacturing cost or size limits for different applications.

All these solutions found by GA and PMA have zero dispersion at telecom wavelength, much lower than the required zero dispersion threshold, $0.01nm^2/m$. In general, PMA appeared to have the equal performance as GA as both of them find zero dispersion. In terms of search speed, GA performs slightly better than PMA because PMA needs to construct GP tree which is a more complex data structure that just numeric values. However, such difference has small impact on the overall speed as the majority of computation is consumed by the waveguide simulator. A single run for PMA in this set of experiments took approximate 28 hours and it was around 26 hours for GA. Both of them ran under the same environment: a quad-core 2.3 GHz machine with 12 GB memory.

4.3 Optimization with Multiple Objectives: Dispersion & Loss

The requirement for dispersion here is exactly the same as for the previous experiments. Propagation loss is added as the second objective. We have evaluated the solutions obtained from the previous experiments in term of loss. None of them could meet that criterion although their dispersion were very low.

The typical multi-objective optimization method, Non-dominated Sorting, could be used to incorporate these two criteria. NSGA-II which has native support for optimizing multiple objectives was introduced into this set of experiments. The fitness measurement for PMA is different from NSGA-II, instead we used the *Weighted Sum Approach* as the dispersion has higher preference than loss in waveguide design. The fitness evaluation is the following:

$$fitness = |Dispersion| \times 100 + |Loss|$$

The weights of dispersion and loss in this fitness measurement were determined empirically. Since the requirement of dispersion is much higher than that of loss, a high weight is assigned to the dispersion property. In this case, it was 100. The choosing of this weight is not limited to just 100. It could be even larger or smaller. The purpose of this weight value is only to specify that dispersion is a much more important feature than loss in waveguide design. Empirically 100 is a more suitable value.

The runtime configuration such as crossover rate, mutation rate, elitism, population size, number of generations were identical to those in the previous set of experiments. The corresponding results for TM mode are shown in Table 5. Similar experiments were conducted in TE mode, and there was no results that met both the dispersion and loss criteria. However, such results are not surprising, since the literature has already stated that it is less likely to form zero-dispersion at telecom wavelength in TE mode when considering more properties (Lamont et al. 2007). Our findings is consistent with the discovery from researchers in optical engineering.

The results presented in Table 5 show that GP is able to find solutions which satisfy both objectives. The results are extracted from the last generation of PMA and NSGA-II. The top 20 results are given here and sorted by the dispersion value. Eleven of the results are from PMA while nine of them are from NSGA-II. It should be noted that the reported loss values are the approximations. The "0" loss values are the direct output from the waveguide simulator which is not able to generate high precision loss value when the loss is very small. However what is certain is that the loss values are much smaller than 0.1 dB/cm.

Table 5 shows that the PMA approach and NSGA-II have very similar performance in terms of minimizing dispersion and loss. However, the average dispersion value from PMA ($\approx 0.00086231 nm^2/m$) is slightly better than that from NSGA-II ($\approx 0.0010853 nm^2/m$). The best solution was found by PMA. The dispersion value of it is only 0.000173335 nm^2/m. The difference between these approaches may be due to the multi-objective fitness measurement. In the Non-dominated Sorting in NSGA-II, all the objectives are treated equally, while in the weighted sum approach in PMA, dispersion is give a much higher weight than than loss. Due to the priority setting, PMA could focus on finding solutions with smaller dispersion value. In our further study, we will investigate the effectiveness of weighted sum approach compared with the standard non-dominated sorting in handling multiple objectives.

5 Conclusions and Future Work

In this paper we studied designing optimal waveguide structures with multiple objectives using evolutionary techniques, namely GA and PMA, a GP-based optimization method. The aim here is not to just find one solution but a set of different solutions. Based on the investigation we conclude the followings: evolutionary search methods GA and GP are suitable for solving this real world problem. They are capable of finding multiple waveguide structures which meet multiple design objectives. By the combination of PMA and weighted sum fitness, and by NSGA-II, we were able to find waveguide structures with dispersion as well as propagation loss much lower than what in the current literature. Additionally little human intervention is required by these methods to generate these satisfactory optical waveguides.

The GP-based PMA method is a suitable method for parametric optimization. As stated in (Pujol & Poli 2008), PMA should be investigated on real-world problems. To our knowledge this is the first time PMA was tested on a real-world application. It is arguably better than or at least equivalent to classical GA in this problem.

In the near future we will combine parametric optimization and geometric design of waveguide into a single framework by using GP, so the structure may contain multiple ridges and may not be rectangular or symmetric. Furthermore we will incorporate more objectives such as maximizing nonlinearity and maintaining a single mode in the evolutionary waveguide design.

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Table 5: Optimizing Dispersion & Loss in TM mode using PMA and NSGA-II

Solutions	Width (μm)	Height (μm)	Etch Depth (μm)	Dispersion (/W/km)	Loss (dB/cm)	Approach
1	1.820	1.029	0.621	0.000173335	0.0	PMA
2	1.629	1.073	0.774	0.000196365	0.0	PMA
3	1.401	1.130	0.940	0.000222729	0.0	NSGA-II
4	1.410	1.069	0.625	0.000224547	0.0	NSGA-II
5	1.301	1.138	0.916	0.000263336	0.0	PMA
6	0.848	1.222	1.218	0.000547884	0.0	PMA
7	0.986	1.198	1.090	0.000647582	0.0	PMA
8	1.998	1.029	0.745	0.00079728	0.0	NSGA-II
9	1.876	1.015	0.544	0.000811825	0.0	PMA
10	1.293	1.120	0.831	0.000911523	0.0	PMA
11	1.361	1.067	0.580	0.000951221	0.0	NSGA-II
12	1.167	1.177	1.041	0.00107001	0.0	PMA
13	1.249	1.082	0.626	0.00110486	0.0	NSGA-II
14	1.841	1.073	0.985	0.00114456	0.0	NSGA-II
15	1.020	0.630	0.452	0.0016485	0.0	PMA
16	0.898	1.134	0.929	0.00171092	0.0	PMA
17	1.049	1.116	0.773	0.00171183	0.0	PMA
18	1.650	1.020	0.458	0.0017688	0.0	NSGA-II
19	1.783	0.938	0.303	0.00177244	0.0	NSGA-II
20	0.782	1.135	1.027	0.00179699	0.0	NSGA-II

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