Image Segmentation on Spiral Architecture

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Abstract

Spiral Architecture is a relatively new and powerful approach to general purpose machine vision system. It contains very useful geometric and algebraic properties. Two algebraic operations, Spiral Addition and Spiral Multiplication, have been defined on it. This paper presents a way to segment the object(s) in an image uniformly by Spiral Multiplication. Namely, a number of analogous small copies of the original object(s) are made during segmentation. An algorithm is also developed in this paper to compute the scaling factor or the number of the small copies, so image segmentation can be done flexibly and quantitatively according to the specific application. The research results are very beneficial to image segmentation in parallel image processing and distributed image processing.

Keywords: Image rotation, image segmentation, distributed image processing

1 Introduction

The algorithm to be presented in this paper is based on a special data structure, Spiral Architecture (Sheridan, Hintz and Moore 1991) which is inspired from anatomical considerations of the primate's vision (Schwartz 1980). From the research about the geometry of the cones on the primate's retina it can be concluded that the cones' distribution has inherent organization and is featured by its potential powerful computation abilities. The cones with the shape of hexagons are arranged in a spiral cluster. This cluster consists of the organizational units of vision. Each unit is a set of seven-hexagon (Sheridan, Hintz and Alexander 2000) as shown in Figure 1.

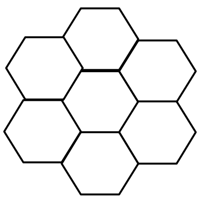


Figure 1. Seven-hexagon unit of vision on Spiral Architecture

In the traditional rectangular image architecture, a set of 3×3 rectangles is used as the unit of vision and each pixel has eight neighbour pixels (See Figure 2). In Spiral Architecture any pixel has only six neighbour pixels which have the same distance to the center hexagon of the seven-hexagon unit of vision. So the Spiral Architecture has the possibility to save time for global and local processing.

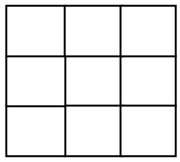


Figure 2. Unit of vision on rectangular image architecture

A natural data structure that emerges from geometric consideration of the distribution of photo receptors on the primate's retina has been called the Spiral Honeycomb Mosaic (SHM) and is presented in details in (Sheridan and Hintz 1999). SHM is made up of the hexagonal lattices, which are identified by a designated positive number individually. The numbered hexagons form the cluster of size 7^n. The hexagons tile the plane in a recursive modular manner along the spiral direction (Alexander 1995). An example of a cluster with size of 7^2 and the corresponding addresses are shown in Figure 3.

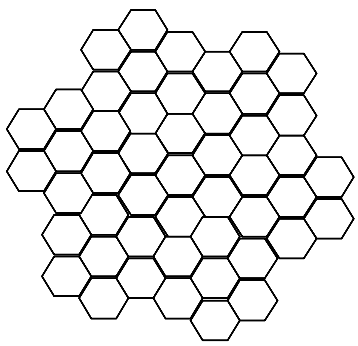


Figure 3. SHM with size of 49

SHM contains very useful geometric and algebraic properties, which can be interpreted in terms of the mathematical object, Euclidean ring. Two algebraic operations have been defined on SHM, Spiral Addition and Spiral Multiplication. After image is projected onto SHM, each pixel on the image is associated with a particular hexagon and its SHM address. Then these two operations mentioned above can be used to define two transformations on SHM address space respectively, which are translation of image and scaling rotation of image. This paper deepens the research work of Spiral Multiplication to achieve uniform image segmentation. Moreover, such segmentation can be measured exactly and quantitatively. It is very useful to distributed image processing (Bharadwaj 2000). By uniform image segmentation, working load can be balanced among all the nodes in the distributed processing system. So system performance is improved very much.

There is no doubt that this algorithm lights a gateway for the application of Spiral Architecture in image processing.

The organization of this paper is as follows. In order to simplify our presentation, the relative knowledge about Spiral Multiplication will be explained briefly in Section 2 accordingly. Section 3 shows a way to compute scaling factor or the number of small copies during image segmentation. The experiment results are demonstrated in Section 4. Conclusion can be seen in Section 5.

2 Spiral Multiplication

SHM is a subset of the complex plane. Spiral Multiplication is an arithmetic operation with closure properties defined on SHM addressing system so that the resulting product will be SHM address in the same finite set on which the operation is performed (Sheridan 1996). In addition, Spiral Multiplication incorporates a special form of modularity.

In order to achieve Spiral Multiplication, a scalar form of Spiral Multiplication is defined in Table I.

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6
2	0	2	3	4	5	6	1
3	0	3	4	5	6	1	2
4	0	4	5	6	1	2	3
5	0	5	6	1	2	3	4
6	0	6	1	2	3	4	5

Table I: Scalar Spiral Multiplication Table

Multiplication of address a by the scalar $\alpha(\alpha \in \{0,1,\cdots,6\})$ is obtained by applying scalar multiplication to the components of a according to the above scalar form, and denoted by,

$$\alpha(a) = (\alpha a_n \;\; \alpha a_{n-1} \;\cdots\; \alpha a_1) \quad \text{where} \\ a = (a_n a_{n-1} \cdots a_1) \text{ for } \forall a_i \in \{0,1,\cdots,6\} \tag{2.1}$$

If the address in Spiral Multiplication is not a scalar, α, but a common address like,

$$b = (b_n b_{n-1} \cdots b_1) \text{ for } \forall b_i \in \{0,1,\cdots,6\} \tag{2.2}$$

then

$$a \times b = \sum_{i=1}^{n} a \times b_i \times 10^{i-1} \tag{2.3}$$

where $\sum$ denotes Spiral Addition and $\times$ denotes Spiral Multiplication. Surely *carry rule* is required in Spiral Addition to handle the addition of numbers composed of more than one digit.

In order to guarantee that all the pixels are still located within the original Spiral area after Spiral Multiplication, a modular multiplication on SHM is defined. Furthermore the transformation through Spiral Multiplication defined on SHM is a bijective mapping. That is each pixel in the original image maps one-to-one to each pixel in the output image after Spiral Multiplication.

Modular Multiplication is shown as follows,

Let p be the product of two elements a, b. That is,

$$p = a \times b, \; a, b \in SHM \tag{2.4}$$

If $p \geq (\text{modulus})$, then
if a is a multiple of 10 map p to

$$\left(p + (p \div (\text{modulus}))\right) \text{mod}(\text{modulus}) \tag{2.5}$$

otherwise, map p to

$$p \,\text{mod}(\text{modulus}), \text{ where} \\ \text{modulus} = 10^n \tag{2.6}$$

Here, it is assumed that the number of hexagon in spiral area is 7^n.

In addition, another point relative to Spiral multiplication is the existence of inverse multiplicative. Given an element $a \in SHM$, there exists an inverse value $b \in SHM$, such that $a \times b = 1$ (Spiral Multiplication), denoted by a^{-1}, i.e., $b = a^{-1}$.

Two cases must be considered to find out the inverse value for one SHM address.

CASE 1 a is not a multiple of 10.

Let us assume $a = (a_n a_{n-1} \cdots a_1)$ for $\forall a_i \in \{0,1,\cdots,6\}$ and the inverse value $b = (b_n b_{n-1} \cdots b_1)$ for $\forall b_i \in \{0,1,\cdots,6\}$. In general, we can get the inverse values for the basic SHM addresses 1, 2, 3, 4, 5 and 6. They are 1, 6, 5, 4, 3 and 2 respectively.

So the inverse value, b, can be constructed successfully by the following formula,

$$\begin{aligned} b_1 &= a_1^{-1} \\ b_2 &= -(a_2 b_1) \times b_1 \\ &\vdots \\ b_n &= (-(\sum_{i=0}^{n-2} a_{n-i} \times b_{i+1})) \times b_1 \end{aligned} \qquad (2.7)$$

CASE 2 a is a multiple of 10, i.e.,

$$\begin{aligned} &a = k \times 10^m \ (m < n) \\ &\text{modulus} = 10^n \end{aligned} \qquad (2.8)$$

We can get k^{-1} by formula (2.7). So,

$$a^{-1} = k^{-1} \times 10^{n-m} \text{ (Spiral Multiplication)} \qquad (2.9)$$

Here we assume that SHM address 0 has no valid inverse value.

3 Scaling Factor Computation While Image Segmentation

After a Spiral image is multiplied by a specific Spiral Address, x, this image will rotate by angle θ which is determined by vector $\overrightarrow{01}$ and vector $\overrightarrow{0X}$ (He, Hintz and Sheridan 1996) (See Figure 4).

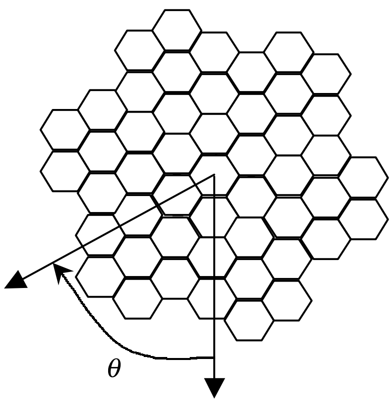


Figure 4. Rotation angle is determined by multiplier

But this rotation is not a pure rotation. It is accompanied by a scaling segmentation. It is disadvantageous to image rotation, but it inspires us with a good idea to segment the objects in an image uniformly. Unfortunately, it is known only how to segment the object into 7^n parts if there are $7^m (m > n)$ pixels on the Spiral Architecture. This can be done by Spiral Multiplication with a multiplier 10^{m-n} directly. There has not been a way yet to compute scaling factor or the number of segments during image segmentation if the multiplier is not a multiple of 10 (See Figure 5).

A novel algorithm to compute the scaling factor is developed as follows for the multiplier being an arbitrary Spiral Address.

STEP 1 Refine Spiral Architecture

In order to segment the objects into any number of parts, the relation between the multiplier and the number of segments after Spiral Multiplication must be found. In this paper, Spiral Architecture is refined and is shown in Figure 6. Here, three parameters are introduced to locate each pixel on Spiral Architecture.

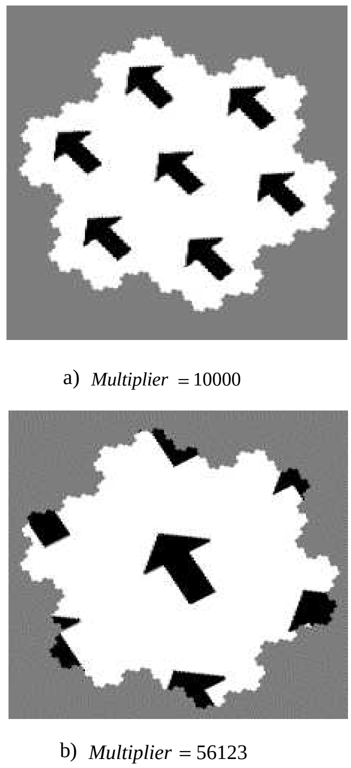


Figure 5. Image segmentation by Spiral Multiplication. Original image is an up-right arrow with 16807 pixels on Spiral Architecture.

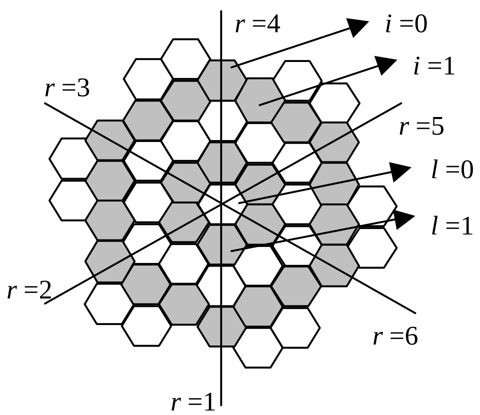


Figure 6. Subdivided Spiral Architecture

The original Spiral Architecture is divided into 6 regions, which is denoted by $r = 1,2,\cdots,6$. In each region, the pixels are grouped in different levels denoted by $l = 0,1,\cdots$ along the radial direction. On each level, each pixel is regarded as an item denoted by i , where $i = 0,1,\cdots,l$ clockwise as shown in Figure 4. So each pixel can be located by these three parameters, (*r, l, i*), uniquely in addition to the Spiral Address on Spiral Architecture.

STEP2 Locate inverse value of multiplier in Figure 6

Suppose the original image is multiplied by Spiral Address x , its corresponding inverse Spiral Address, $y = x^{-1}$, can be obtained by the way mentioned in section 2. Then the location parameters, (r,l,i) of y can be gotten according to STEP 1.

STEP3 Compute scaling factor

Our key contribution is to develop a formula for computing the scaling factor during image segmentation on Spiral Architecture. This formula is shown,

$$\begin{aligned} &Scale^{-1}_{(r,l,i)} = [l^2 - i(l-1) + i(i-1)] \qquad where, \\ &r = 1,2,...,6; \\ &l = 1,2,...; \text{ and} \\ &i = 0,1,...,l. \end{aligned} \tag{3.1}$$

It is found that scale value is only determined by the parameters, l and i . That means the final scaling factor is determined by the level number and the item number of the inverse value of multiplier. The only differences among the images derived from Spiral Multiplication with the different multipliers are the rotation angles. The angle difference is the multiple of 60 degrees.

Using the above algorithm with Spiral Multiplication, an image can be segmented as required to many parts which are the condensed copies of the original image on Spiral Architecture. We can also uniformly separate the original image to sub-images of certain size based on the requirement of precision and the capacity of processing nodes on the network for distributed image processing.

The experiment results are shown in Section 4.

4 Experiment Results

As a simplified illustration of our algorithm without loss of generality, an image containing an up-right arrow and an image containing a toy duck (See Figure 7) are used here. There are totally $16807 = 7^5$ hexagonal pixels in the Spiral Architecture area individually.

Suppose that the original image is multiplied by Spiral Address 56123. According to Spiral Multiplication principle the inverse value of 56123 is 15. On the refined Spiral Architecture, Spiral Address 15 is located by the parameter $(1,2,1)$ (See Figure 8). Then the scaling factor is *1/3* by formula (3.1) (See Figure 9).

In Figure 9, we find that three near copies are created after the above multiplication. This is due to the fact that each copy results from a unique sampling of the input image. Each sample is mutually exclusive and the collection of all such samples represents a partitioning of the input image. As the scaling in effect represents the viewing of the image at a lower resolution, each copy has less information.

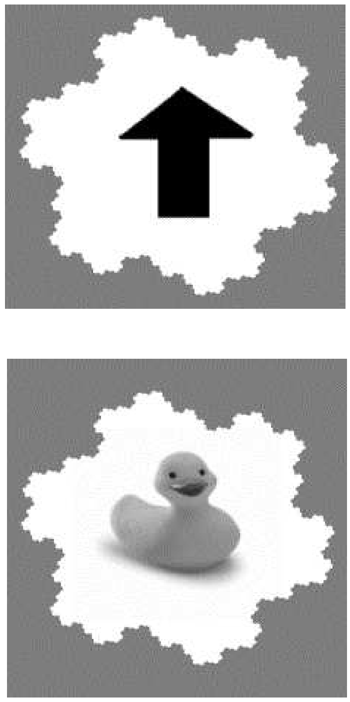


Figure 7. Up-right arrow and toy duck

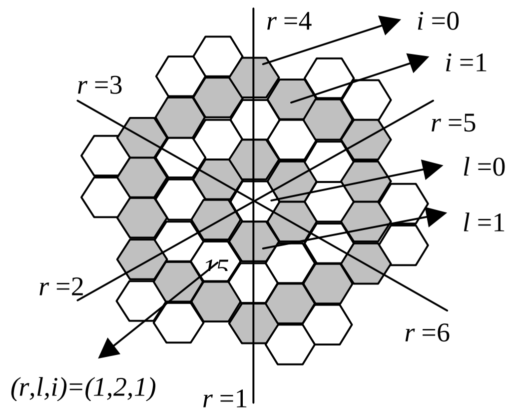
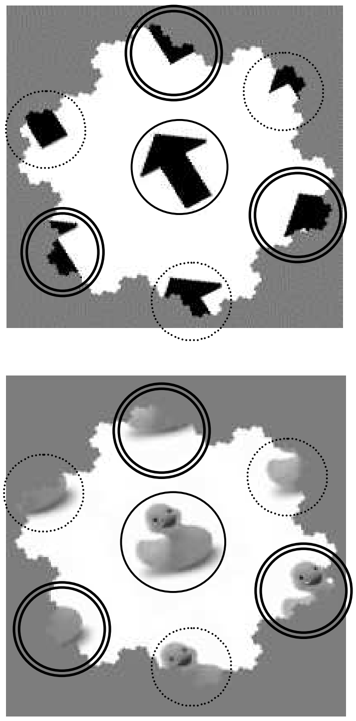


Figure 8. Parameter, (r,l,i) , of inverse value of multiplier, 15

However, as none of the individual light intensities have been altered in any way, the scaled image still holds all of the information contained in the original. This means that the computational complexity has been both reduced and nicely partitioned without giving away any information if distributed processing system is implemented here. With the help of the algorithm mentioned above for computing scaling factor we can partition the original image correctly and quantitatively on Spiral Architecture based on the practical distributed system performance and detail requirements in the real situation.

It is also found that image rotation accompanies with image segmentation, but this kind of rotating effect will not affect image processing if image segmentation here is for object recognition as the final goal. Finding object representation invariant to affine transformation can avoid such rotating effects.



Near copy 1 —

Near copy 2 ═

Near copy 3 - -

Figure 9. Scaling factor after Spiral Multiplication by Spiral Address 56123

REMARK In order to make Spiral Architecture practically workable on the existing image capture device, mimic Spiral Architecture is used in the research work (He 1999). Due to a few differences between real Spiral Architecture and mimic Spiral Architecture, a little distortion is introduced into the image during image rotation (See Figure 7, Figure 9). But it does not affect the theoretical research about Spiral Architecture. Surely, this distortion will be resolved with the development of image capture hardware device.

5 Conclusion

This paper presents the deep research work about Spiral Multiplication on Spiral Architecture. A new way is developed to measure image segmentation quantitatively on Spiral Architecture by Spiral Multiplication. From the experimental results we see the objective is achieved successfully. It successfully improves the Spiral Architecture's usage in image processing, and especially in image segmentation for distributed image processing.

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